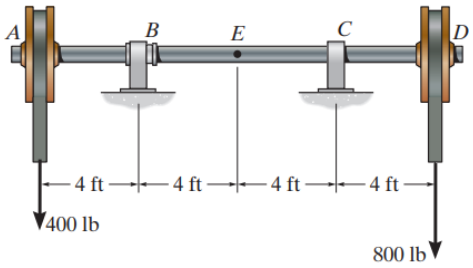
Problem 1-1

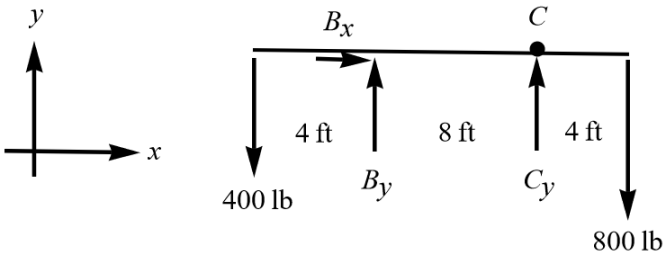
The shaft is supported by a smooth thrust bearing at B and a journal bearing at C. Determine the resultant internal loadings acting on the cross section at E.



Prob. 1–1

Solution

According to Table 1-1 on page 5, the journal bearing at C prevents motion in the vertical direction, and the thrust bearing at B prevents motion in the horizontal and vertical directions. As a result, there are three unknown forces in the free-body diagram of the shaft shown below.



Use Newton's second law to obtain two equations involving the unknown forces.

$$\sum \mathbf{F} = m\mathbf{a} \quad \Rightarrow \quad \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \end{cases} \quad \rightarrow \quad \begin{cases} B_x = 0 \\ -400 + B_y + C_y - 800 = 0 \end{cases}$$

Take the sum of the moments about point C to get a third equation involving only B_y.

$$\circlearrowleft^{+} \sum \mathbf{M}_C = \mathbf{0} \quad \Rightarrow \quad (400 \text{ lb})(12 \text{ ft}) - B_y(8 \text{ ft}) - (800 \text{ lb})(4 \text{ ft}) = 0$$

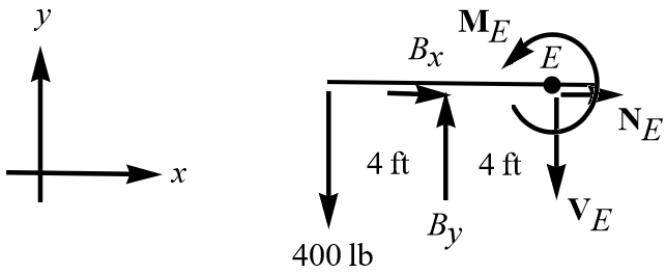
Solving these three equations gives

$$B_x = 0$$

$$B_y = 200 \text{ lb}$$

$$C_y = 1000 \text{ lb}.$$

Now that the reactions at B and C are known, the internal loadings at E can be determined using the method of sections.



Find them with the equations of equilibrium.

$$\sum F_x = B_x + N_E = 0$$

$$\sum F_y = -400 + B_y - V_E = 0$$

$$\circlearrowleft^+ \sum (\text{Moments about } E) = M_E - B_y(4 \text{ ft}) + (400 \text{ lb})(8 \text{ ft}) = 0$$

Plug in the reactions

$$N_E = 0$$

$$-400 + 200 - V_E = 0$$

$$M_E - (200 \text{ lb})(4 \text{ ft}) + (400 \text{ lb})(8 \text{ ft}) = 0$$

and solve the system of equations for N_E, V_E, and M_E.

$$N_E = 0$$

$$V_E = -200 \text{ lb}$$

$$M_E = -2400 \text{ lb} \cdot \text{ft}$$